Value Based - Part 2

Deep Reinforcement Learning

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Courtesy: Some slides are adopted from CS 285 Berkeley, and CS 234 Stanford, and Pieter Abbeel's compact series on RL.

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Disadvantages of Monte-Carlo Learning

- We have seen MC algorithms can be used to learn value predictions
- But when episodes are long, learning can be slow
  - ☑ we have to wait until an episode ends before we can learn…
  - ☑ return can have high variance
    - Which one is more? First-visit or every-visit
- Are there alternatives? (Spoiler: yes)

Monte Carlo Control

Monte-Carlo Control

Algorithm:

Repeat:

Sample episode $1, \dots, k, \dots,$ **using** $\pi: \{S_1, A_1, R_2, \dots, S_T\} \sim \pi$

For each state S_t and action A_t in the episode:

$$q(S_t, A_t) \leftarrow q(S_t, A_t) + \alpha_t \big(G_t - q(S_t, A_t)\big)$$

e.g.,

$$\alpha_t = \frac{1}{N(S_t, A_t)} \quad of \quad \alpha_t = 1/k$$

Improve policy based on new action-value function

$$\pi^{new}(s) = \underset{a \in A}{\operatorname{argmax}}\, q(s, a)$$

Any issue?

- Let's consider this example:
- Discount = 1, start in state H.

A	B	C	D	E	F	G	H	I	J
r=10	→	→	→	→	→	↑	→	→	r=1

Epsilon Greedy Policy

- Simple idea to balance exploration and achieving rewards
- Let $|A|$ be the number of actions
- Then an ϵ-greedy policy w.r.t a state action value $Q(s,a)$ is

$$\pi(a|s) =$$

- ☑ $\underset{a}{\mathrm{arg}max} Q(s,a),\ w.\mathrm{P}rob\ 1-\epsilon+\frac{\epsilon}{|A|}$
- ☑ $a' \neq \mathrm{arg}max Q(s,a)\, w.\mathrm{P}rob\ \frac{\epsilon}{|A|}$

Does this hurt improvement?

Theorem

For any ϵ-greedy policy π_i, the ϵ-greedy policy w.r.t. Q^{π_i}, π_{i+1} is a monotonic improvement $V^{\pi_{i+1}} \geq V^{\pi_i}$

Monte-Carlo Control (done right)

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Improve policy based on new action-value function

$$\pi(a|s) =$$

$$\underset{a}{\operatorname{argmax}} Q(s,a), w.\text{P}rob \;\; 1 - \epsilon + \frac{\epsilon}{|A|}$$

$$a' \neq \operatorname{argmax} Q(s,a)\, w.\text{P}rob \;\; \frac{\epsilon}{|A|}$$

Disadvantages of MC Learning

- We have seen MC algorithms can be used to learn value predictions
- But when episodes are long, learning can be slow
  - ☑ ...we have to wait until an episode ends before we can learn
  - ☑ ...return can have high variance
- Are there alternatives? (Yes)

Temporal Difference Learning

Prediction

TD Overview

- TD methods learn directly from episodes of experience
- TD is model-free: no knowledge of MDP transitions / rewards
- TD learns from incomplete episodes, by bootstrapping
- TD updates a guess towards a guess

TD Learning by Sampling Bellman Equations

- Bellman update equations:

$$v_{k+1}(s) = \mathbb{E}[R_{t+1} + \gamma v_k(S_{t+1}) \mid S_t = s, A_t \sim \pi(S_t)]$$

- We can sample this!

$$v_{t+1}(S_t) = R_{t+1} + \gamma v_t(S_{t+1})$$

- Samples could be averaged, in a similar way to MC:

$$v_{t+1}(S_t) \leftarrow v_t(S_t) + \alpha \left(\underbrace{\underbrace{R_{t+1} + \gamma v_t(S_{t+1})}_{\text{target}} - v_t(S_t)}_{\text{temporal difference error } \delta_t} \right)$$

Temporal Difference Learning

- Prediction setting: learn v_π online from experience under policy π
- Monte Carlo
  - Update value $v_n(S_t)$ towards sampled return G_t

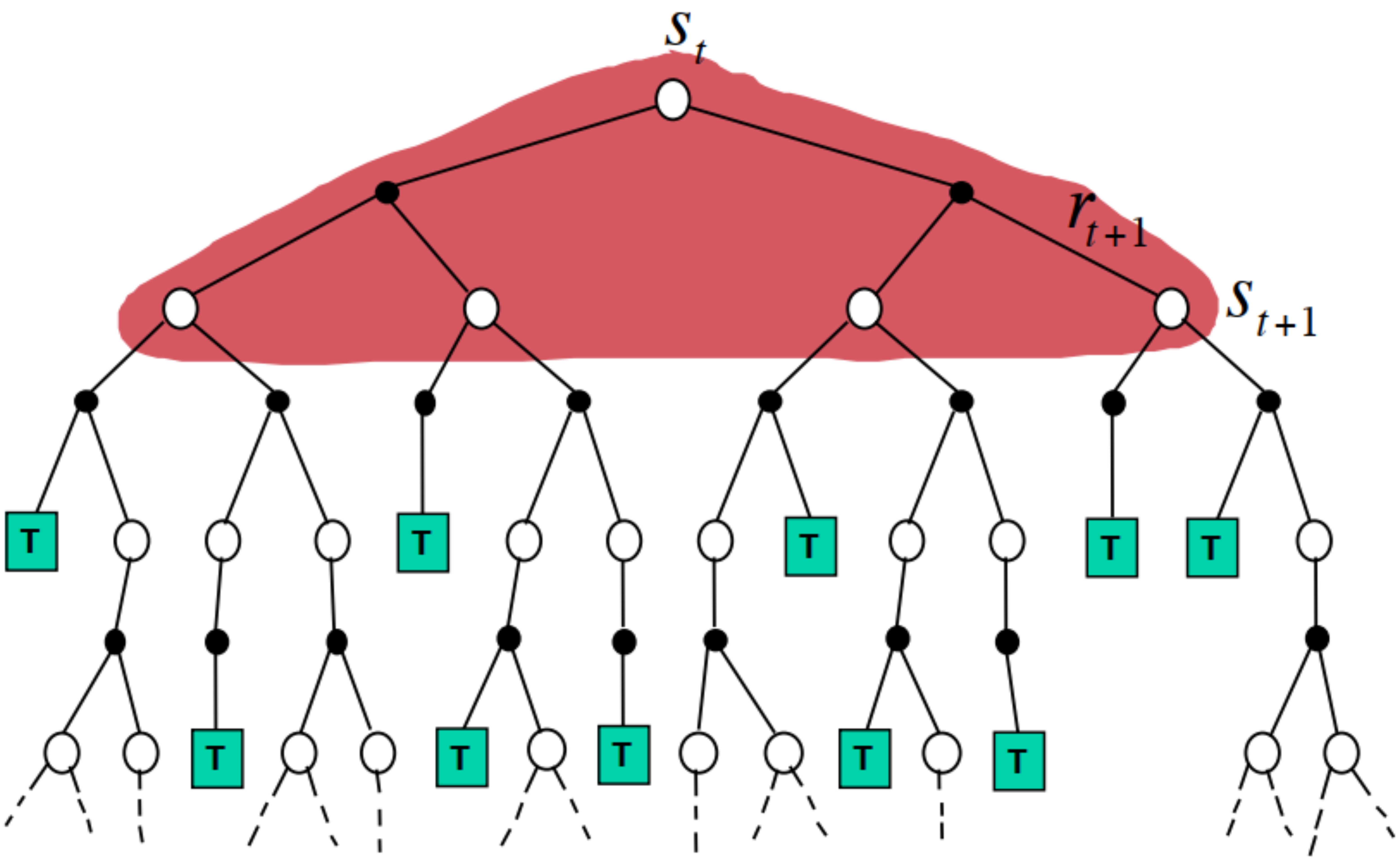
$$v_{n+1}(S_t) = v_n(S_t) + \alpha(G_t - v_n(S_t))$$

- TD Learning
  - Update value $V_t(S_t)$ towards estimated return $R_{t+1} + \gamma v(S_{t+1})$

$$v_{t+1}(S_t) \leftarrow v_t(S_t) + \alpha\left(\overbrace{\underbrace{R_{t+1} + \gamma v_t(S_{t+1})}_{\text{target}} - v_t(S_t)}^{\text{TD error}}\right)$$

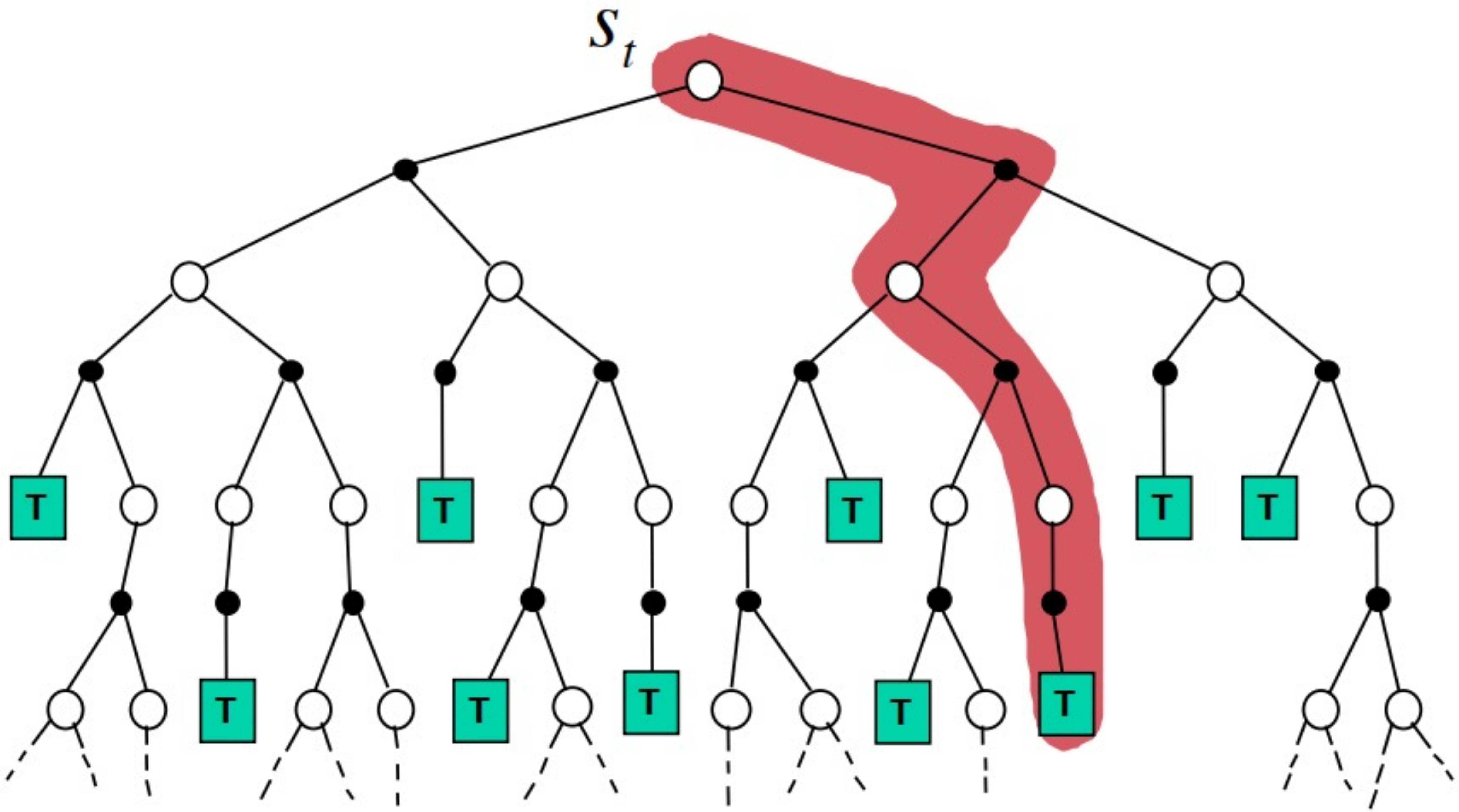
Backup (Dynamic Programming)

$$v(S_t) \leftarrow \mathbb{E}[\, R_{t+1} + \gamma v(S_{t+1}) \mid A_t \sim \pi(S_t) \,]$$



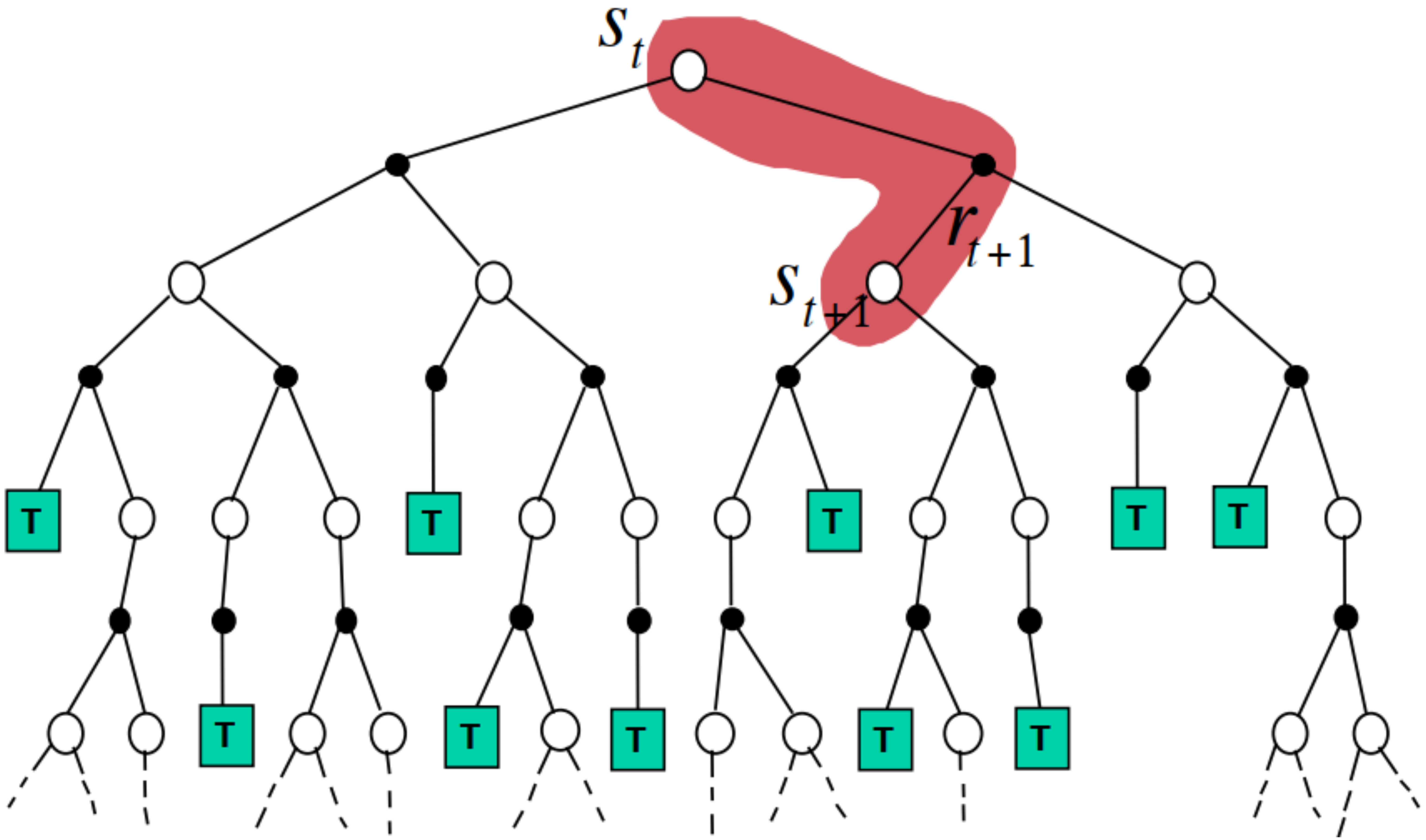
Backup (Monte Carlo)

$$v(S_t) \leftarrow v(S_t) + \alpha(G_t - v(S_t))$$



Backup (Temporal Difference)

$$v(S_t) \leftarrow v(S_t) + \alpha(R_{t+1} + \gamma v(S_{t+1}) - v(s_t))$$



Bootstrapping and Sampling

- Bootstrapping: update involves an estimate
  - MC does not bootstrap
  - DP bootstraps
  - TD bootstraps
- Sampling: update samples an expectation
  - MC samples
  - DP does not sample
  - TD samples

TD Learning for action values

- We can apply the same idea to action values
- Temporal-difference learning for action values:

☑ Update value $q_t(S_t, A_t)$ towards estimated return $R_{t+1} + \gamma q(S_{t+1}, A_{t+1})$

$$q_{t+1}(S_t, A_t) \leftarrow q_t(S_t, A_t) + \alpha \left(\overbrace{\underbrace{R_{t+1} + \gamma q_t(S_{t+1}, A_{t+1})}_{\text{target}} - q_t(S_t, A_t)}^{\text{TD error}} \right)$$

TD vs. MC

- TD can learn before knowing the final outcome
  - TD can learn online after every step
  - MC must wait until end of episode before return is known
- TD can learn without the final outcome
  - MC must wait until end of episode before return is known
  - MC can only learn from complete sequences
  - TD works in continuing (non-terminating) environments
  - MC only works for episodic (terminating) environments
- TD is independent of the temporal span of the prediction
  - TD can learn from single transitions
  - MC must store all predictions (or states) to update at the end of an episode
- TD needs reasonable value estimates

Temporal Difference Learning

Control

SARSA Algorithm for On-Policy Control

Algorithm:

Initialize $Q(s, a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$, **arbitrarily, and** $Q(terminal\!-\!state, \cdot) = 0$

Repeat (for each episode):

  Initialize S

  Choose A **from** S **using policy derived from** Q **(e.g.,** ϵ**-greedy)**

  Repeat (for each step of episode):

    Take action A**, observe** R, S'

    Choose A' **from** S' **using policy derived from** Q **(e.g.,** ϵ**-greedy)**

    $Q(S, A) \leftarrow Q(S, A) + \alpha[R + \gamma Q(S', A') - Q(S, A)]$

    $S \leftarrow S'; A \leftarrow A';$

  until S **is terminal**

Off-Policy TD and Q-Learning

On and Off-Policy Learning

- On-policy learning
  - "Learn on the job“
  - Learn about policy π from experience sampled from π
- Off-policy learning
  - "Look over someone’s shoulder“
  - Learn about policy π from experience sampled from μ

Off-Policy Learning

- Evaluate target policy $\pi(a|s)$ to compute $v_\pi(s)$ or $q_\pi(s,a)$
- While using behavior policy $\mu(a,s)$ to generate actions
- Why is this important?
  - Learn from observing humans or other agents (e.g., from logged data)
  - Re-use experience from old policies (e.g., from your own past experience)
  - Learn about multiple policies while following one policy
  - Learn about greedy policy while following exploratory policy

Q-Learning

- Q-learning estimates the value of the greedy policy

$$q_{t+1}(s,a) = q_t(S_t, A_t) + \alpha_t \left(R_{t+1} + \gamma \max_{a'} q_t(S_{t+1}, a') - q_t(S_t, A_t) \right)$$

- Acting greedy all the time would not explore sufficiently

Theorem

Q-learning control converges to the optimal action-value function, $q \to q^*$, as long as we take each action in each state infinitely often.

- Note: no need for greedy behavior!
- Works for any policy that eventually selects all actions sufficiently often

Q-Learning for Off-Policy Control

Algorithm:

Initialize $Q(s,a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$, **arbitrarily, and** $Q(terminal{-}state, \cdot) = 0$

Repeat (for each episode):

  Initialize S

  Repeat (for each step of episode):

    Choose A **from** S **using policy derived from** Q **(e.g.,** ϵ**-greedy)**

    Take action A**, observe** R, S'

    $Q(S,A) \leftarrow Q(S,A) + \alpha[R + \gamma max_a Q(S',a) - Q(S,A)]$

    $S \leftarrow S'$;

  until S **is terminal**